

## METHODOLOGY OF MATHEMATICAL ANALYSIS IN POWER NETWORK

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**Abstract:** Power distribution network analysis is taken into account. Based on correlation coefficient authors establish methodology of mathematical analysis useful in finding substations bear responsibility for power stoppage. Also methodology of risk assessment will be carried out.

**Keywords:** Correlation Critical Clusters, distribution network, probability of power stoppage.

### Introduction

The power network can be divided into three subsystems which are:

1. The main transmission network – includes generating facilities and the transmission network which is designed to carry large quantities of energy over long distances;
2. The sub-transmission network, which is designed to carry smaller quantities of energy over shorter distances;
3. The distribution network where electricity is transmitted locally at medium voltage and low voltage.

Each network can be divided into smaller subsystems which can be analyzed separately. Presented methodology can be applied at any stage of accuracy. Therefore, one can analyze any network as long it can be treated as a subsystem of the power network.

Proposed methodology is based on results of further research concerning methodology of determining risk measure in network diagram of production process [1].

Presented methodology allows to determine failure of power supply in power network based on factor characterized network state. Using correlation coefficient factor authors can determine risk of network failure and its probability.

### 1. Power distribution network as a digraph

Analysis of power distribution is based on graph theory [2]. Simple graph is a (usually finite) set of mouldings (nodes)  $V$  and set of unordered pairs of distinct elements of  $V$  called edges (nodes). Edges can also be given a direction yielding a directed graph (digraph) [2]. Therefore, power distribution network can be displayed as a digraph, where transmission stations are presented as vertices (nodes) and connections between them presents transmission lines. Example of digraph is presented on Figure 1.

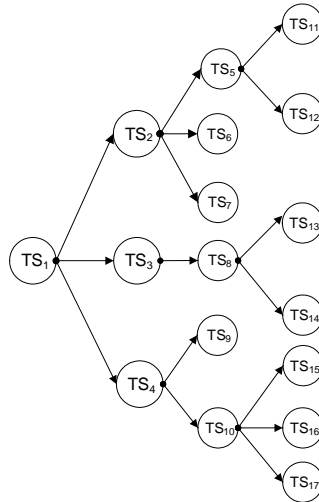


Figure 1. 110 kV power distribution network

There are many factors which are describing power distribution network like: electric power<sup>1</sup>, voltage<sup>2</sup>, electric resistance [3]<sup>3,4</sup>. In fact one of listed parameters is enough to run research. For purposes of this article it will be electric power factor. However, one can choose any factor which efficiently describes network characteristics. Energetic companies aspire to achieve position allowing to be flexible to predictable and unpredictable precinct changes [4]. Thus presented methodology is alternative tool allowing to achieve this aim.

To conduct research one must determine each device characteristic in our case it will be transmission stations (*TS*), we assume that all *TS* is  $110kV/30kV/15kV$  station. Summing up we will introduce methodology of determine failure of power supply in power network compounded of 17 transmission stations with technical parameters:  $110kV/30kV/15kV$ . There is a necessity of carrying out measurements of chosen parameter in the network. Authors suggest to take statistics which includes at least one month state of network with half an hour frequency [5].

## 2. Correlation coefficient and variance

Model defined leaning on hints form chapter 1 allow to conduct further research. To determine failure or power supply in our power network correlation coefficient will be used.

<sup>1</sup> Electric power is defined as the rate at which electrical energy is transferred by an electric circuit. The SI unit of power is the watt.

<sup>2</sup> Voltage (sometimes also called electric or electrical tension) is the difference of electrical potential between two points of an electrical or electronic circuit, expressed in volts. It measures the potential energy of an electric field to cause an electric current in an electrical conductor. Depending on the difference of electrical potential it is called extra low voltage, low voltage, high voltage or extra high voltage. Specifically Voltage is equal to energy per unit charge.

<sup>3</sup> Electrical resistance is a ratio of the degree to which an object opposes an electric current through it, measured in ohms. Its reciprocal quantity is electrical conductance measured in Siemens.

<sup>4</sup> One can list more advanced factors describing network like: resistance low, resistance coupling, cyclic voltage variation, nominal rating (MVA), or voltage –current characteristic.

Table 1. Correlation matrix of transformation stations

	TS <sub>1</sub>	TS <sub>2</sub>	TS <sub>3</sub>	TS <sub>4</sub>	TS <sub>5</sub>	TS <sub>6</sub>	TS <sub>7</sub>	TS <sub>8</sub>	TS <sub>9</sub>	TS <sub>10</sub>	TS <sub>11</sub>	TS <sub>12</sub>	TS <sub>13</sub>	TS <sub>14</sub>	TS <sub>15</sub>	TS <sub>16</sub>	TS <sub>17</sub>
TS <sub>1</sub>	1	0,79	0,87	0,09	0,80	0,05	0,83	0,87	0,76	0,73	0,68	0,35	0,84	0,74	0,72	0,75	0,78
TS <sub>2</sub>	0,79	1	0,85	0,09	0,78	0,03	0,75	0,88	0,72	0,71	0,65	0,33	0,83	0,70	0,65	0,66	0,79
TS <sub>3</sub>	0,87	0,85	1	0,03	0,87	-0,14	0,82	0,90	0,76	0,78	0,69	0,37	0,86	0,76	0,72	0,73	0,82
TS <sub>4</sub>	0,09	0,09	0,03	1	0,04	0,25	0,08	0,10	0,03	0,09	0,03	0,06	0,06	0,00	0,09	0,04	0,07
TS <sub>5</sub>	0,80	0,78	0,87	0,04	1	-0,15	0,79	0,81	0,74	0,80	0,82	0,38	0,87	0,85	0,73	0,83	0,79
TS <sub>6</sub>	0,05	0,03	-0,14	0,25	-0,15	1	0,10	0,05	0,04	0,03	0,01	0,00	0,02	-0,03	0,16	0,03	0,05
TS <sub>7</sub>	0,83	0,75	0,82	0,08	0,79	0,10	1	0,84	0,76	0,74	0,73	0,35	0,81	0,74	0,71	0,77	0,75
TS <sub>8</sub>	0,87	0,88	0,90	0,10	0,81	0,05	0,84	1	0,73	0,76	0,66	0,36	0,83	0,72	0,72	0,69	0,82
TS <sub>9</sub>	0,76	0,72	0,76	0,03	0,74	0,04	0,76	0,73	1	0,54	0,75	0,28	0,81	0,80	0,68	0,77	0,77
TS <sub>10</sub>	0,73	0,71	0,78	0,09	0,80	0,03	0,74	0,76	0,54	1	0,70	0,31	0,76	0,72	0,70	0,69	0,75
TS <sub>11</sub>	0,68	0,65	0,69	0,03	0,82	0,01	0,73	0,66	0,75	0,70	1	0,35	0,77	0,83	0,66	0,84	0,70
TS <sub>12</sub>	0,35	0,33	0,37	0,06	0,38	0,00	0,35	0,36	0,28	0,31	0,35	1	0,31	0,35	0,26	0,35	0,34
TS <sub>13</sub>	0,84	0,83	0,86	0,06	0,87	0,02	0,81	0,83	0,81	0,76	0,77	0,31	1	0,83	0,75	0,82	0,82
TS <sub>14</sub>	0,74	0,70	0,76	0,00	0,85	-0,03	0,74	0,72	0,80	0,72	0,83	0,35	0,83	1	0,77	0,88	0,79
TS <sub>15</sub>	0,72	0,65	0,72	0,09	0,73	0,16	0,71	0,72	0,68	0,70	0,66	0,26	0,75	0,77	1	0,73	0,78
TS <sub>16</sub>	0,75	0,66	0,73	0,04	0,83	0,03	0,77	0,69	0,77	0,69	0,84	0,35	0,82	0,88	0,73	1	0,72
TS <sub>17</sub>	0,78	0,79	0,82	0,07	0,79	0,05	0,75	0,82	0,77	0,75	0,70	0,34	0,82	0,79	0,78	0,72	1

Correlation coefficient [6]<sup>5</sup> is a term meaning mutual connection, correlation of some events or objects which are seen as random variable. In probability theory and statistics correlation is usually understand as a linear relation of random variables, and the measure of this relation is Pearson’s correlation which formula presents:

$$r_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{X})(y_i - \bar{Y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{X})^2 (y_i - \bar{Y})^2}} \tag{1}$$

<sup>5</sup> It might be useful to mention there are also other correlation measures like for instance: Spearman's rank correlation coefficient which does not require the assumption that the relationship between the variables is linear.

where

$x_i$  and  $y_i$  are value of random variables of continues distributions ( $i=1,2,3,\dots,n$ ),  $\bar{X}$  and  $\bar{Y}$  are the estimators of the expected values  $E(X)$  and  $E(Y)$  respectively values of continues. Correlation coefficient interval  $KOR \in [-1;1]$ , where as amount is closer to unity the correlation stronger.[

In order to calculate the value of correlation coefficient for analyzed network one may use computer software. In result of calculations we get correlation matrix which is square and symmetric matrix<sup>6</sup>. Therefore, our matrix is size  $17 \times 17$ . It presents correlation between whole nodes digraph. There is a correlation matrix in Table 1.

### 3. Correlation Critical Clusters

The main criterion which measures the influence between nodes is correlation coefficient. It is important to make a note that chosen coefficient presents relations between distant nodes (not connected directly with each other) which determine the new concept of presented method. Distant nodes affect to other distant one's causing threat in current delivery. For instance let's have a look at node  $TS_2$  and node  $TS_8$  the correlation coefficient is  $0,88$  which means that influence of these two nodes is very high. Although, these nodes are distant from each other (cf. with Figure 1). This observation allows to present influence between nodes in other way than it take place in power distribution network. Figure 2 presents way of determination of correlation in the network.

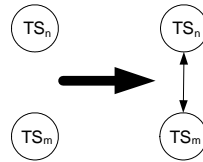


Figure 2. Correlation coefficient between two distant nodes  $n$  and  $m$

Therefore, based on correlation coefficient the new subsets have come into existence [1]:

1. The Correlation Critical Cluster is a subset of nodes with the interval  $(1; 0,9]$  of correlation coefficient which is not a continuous path. It creates a subset of all nodes with the highest correlation coefficient interval.
2. The Correlation Sub-Critical Cluster which is a subset of nodes with the interval  $(0,9; 0,7]$  of correlation coefficient.
3. The Correlation Cluster is a subset of nodes with correlation coefficient lower than  $0,7$  which can be analyzed whenever the need arises.

In order to clarify abstained results one must adopt one of taxonomy methods [7] on the correlation matrix. Thanks to that Correlation Critical Clusters will get grouped together. In another word, as a result of applying taxonomy method we get clear correlation matrix with marked Correlation Critical Clusters. Table 3 present them (cf. with Table 2).

<sup>6</sup> Ibid. Matrix is a rectangular table of elements (or entries), which may be any abstract quantities that can be added and multiplied. More precisely we will say: an  $m \times n$  where  $(m, n \in \mathbb{R})$  matrix  $A$  is a function  $A : \{1, 2, \dots, m\} \times \{1, 2, \dots, n\} \rightarrow S$ , where  $S$  is a non-empty set. A square matrix is a matrix which has the same number of rows and columns. The set of all square  $n$ -by- $n$  matrices, together with matrix addition and matrix multiplication is a ring. Symmetric matrix is a square matrix,  $A$ , that is equal to its transpose  $A = A^T$ .

Table 2. Correlation Critical Clusters

	TS <sub>1</sub>	TS <sub>8</sub>	TS <sub>3</sub>	TS <sub>5</sub>	TS <sub>13</sub>	TS <sub>14</sub>	TS <sub>16</sub>	TS <sub>11</sub>	TS <sub>9</sub>	TS <sub>17</sub>	TS <sub>2</sub>	TS <sub>7</sub>	TS <sub>10</sub>	TS <sub>15</sub>	TS <sub>12</sub>	TS <sub>4</sub>	TS <sub>6</sub>
TS <sub>1</sub>	1	0,87	0,87	0,8	0,84	0,74	0,75	0,68	0,76	0,78	0,79	0,83	0,73	0,72	0,35	0,09	0,05
TS <sub>8</sub>	0,87	1	0,9	0,81	0,83	0,72	0,69	0,66	0,73	0,82	0,88	0,84	0,76	0,72	0,36	0,1	0,05
TS <sub>3</sub>	0,87	0,9	1	0,87	0,86	0,76	0,73	0,69	0,76	0,82	0,85	0,82	0,78	0,72	0,37	0,03	-0,1
TS <sub>5</sub>	0,8	0,81	0,87	1	0,87	0,85	0,83	0,82	0,74	0,79	0,78	0,79	0,8	0,73	0,38	0,04	-0,2
TS <sub>13</sub>	0,84	0,83	0,86	0,87	1	0,83	0,82	0,77	0,81	0,82	0,83	0,81	0,76	0,75	0,31	0,06	0,02
TS <sub>14</sub>	0,74	0,72	0,76	0,85	0,83	1	0,88	0,83	0,8	0,79	0,7	0,74	0,72	0,77	0,35	0	-0,03
TS <sub>16</sub>	0,75	0,69	0,73	0,83	0,82	0,88	1	0,84	0,77	0,72	0,66	0,77	0,69	0,73	0,35	0,04	0,03
TS <sub>11</sub>	0,68	0,66	0,69	0,82	0,77	0,83	0,84	1	0,75	0,7	0,65	0,73	0,7	0,66	0,35	0,03	0,01
TS <sub>9</sub>	0,76	0,73	0,76	0,74	0,81	0,8	0,77	0,75	1	0,77	0,72	0,76	0,54	0,68	0,28	0,03	0,04
TS <sub>17</sub>	0,78	0,82	0,82	0,79	0,82	0,79	0,72	0,7	0,77	1	0,79	0,75	0,75	0,78	0,34	0,07	0,05
TS <sub>2</sub>	0,79	0,88	0,85	0,78	0,83	0,7	0,66	0,65	0,72	0,79	1	0,75	0,71	0,65	0,33	0,09	0,03
TS <sub>7</sub>	0,83	0,84	0,82	0,79	0,81	0,74	0,77	0,73	0,76	0,75	0,75	1	0,74	0,71	0,35	0,08	0,1
TS <sub>10</sub>	0,73	0,76	0,78	0,8	0,76	0,72	0,69	0,7	0,54	0,75	0,71	0,74	1	0,7	0,31	0,09	0,03
TS <sub>15</sub>	0,72	0,72	0,72	0,73	0,75	0,77	0,73	0,66	0,68	0,78	0,65	0,71	0,7	1	0,26	0,09	0,16
TS <sub>12</sub>	0,35	0,36	0,37	0,38	0,31	0,35	0,35	0,35	0,28	0,34	0,33	0,35	0,31	0,26	1	0,06	0
TS <sub>4</sub>	0,09	0,1	0,03	0,04	0,06	0	0,04	0,03	0,03	0,07	0,09	0,08	0,09	0,09	0,06	1	0,25
TS <sub>6</sub>	0,05	0,05	-0,14	-0,15	0,02	-0,03	0,03	0,01	0,04	0,05	0,03	0,1	0,03	0,16	0	0,25	1

where:

(1;0,9]
(0,9;0,7]
(0,7;0]

As it is shown in analyzed network there are no Correlation Critical Clusters. However, there is one Sub-Critical Cluster which is called diffused cluster and in this case is composed of 12 nodes (TS<sub>8</sub>; TS<sub>3</sub>; TS<sub>5</sub>; TS<sub>13</sub>; TS<sub>14</sub>; TS<sub>16</sub>; TS<sub>11</sub>; TS<sub>9</sub>; TS<sub>17</sub>; TS<sub>2</sub>; TS<sub>7</sub>; TS<sub>10</sub>; TS<sub>15</sub>).

The established results show how complex relations exist between nodes in network composed of 17 transmission stations. It is clearly that those correlations are important for network supervision persons. The last step is to determine the probability of power stoppage in analyzed network.

#### 4. Probability of power stoppage

Estimation of the probability power stoppage in given power network is conducted based on variance of diffused cluster. Authors used the following variance formula [8]:

$$S_K^2 = \sum_{i,j \in K} \rho_{ij} S_i S_j w_i w_j \quad (2)$$

where

$\rho_{ij}$  is a correlation coefficient;  $S_i, S_j$  are power standard deviations of individual nodes,  $w_i, w_j$  balance of individual nodes in risk of entire cluster and  $K$  is an index set of subset network elements.

As reader's notice all equation elements have been already estimated. Therefore, by putting the values into the equation we get the risk measure for the power network: 0,29 this means that probability of appearance power stoppage is 29%.

#### 5. Summary

Power network is complex system which is used to transfer energy to end users. Reliability of this system depend on many factors, also many methods are applied in energetics to avoid the power stoppage. Authors using well known correlation coefficient demonstrate the way of using it to describe all unknown factors which have influence on network condition. Presented method allows to prevent critical situations in energetics and can support process of supervision on these systems.

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