MODELLING OF AUTOMATISED LOADING PROCESS OF WAREHOUSES

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Abstract: This paper proposes an integrated mathematical model for the optimisation of a special type of materials handling system. The design and improvement of materials handling and logistic systems is not essentially different from the development of any other engineering systems. The mathematical modelling is gaining more and more importance, because of the requirement of optimised systems. As the investment and operation costs of automatic materials handling systems is rather high, it is very important to use sophisticated methods to model and design the systems. Within the frame of this paper the author describes a mathematical model of a special materials handling system with automatic loading machine. The cost based objective function includes the connections among system parameters. Because of this fact the problem is NP-hard (however the original cost function is convex) and the calculation of the optimal system parameters is possible by the aid of metaheuristic methods.

Keywords: harmony search, materials handling, mathematical modelling, objective function, optimisation

Introduction

Definite methods were available for analysing and optimising materials handling systems and processes in the late 60s [1.]. The development of system engineering leaded to the appearance of new methods and techniques to support the design of complex systems. Not only technical, but also economical, ecological and ergonomic aspects are taken into consideration [2.]. The "how-to" systematic, and methodical approach is very important through the whole analysis and development process [3.]. The design of materials handling systems is very important in each field of industry and services [4-7.], so the quality of mathematical modelling is one of the most important factor from the point of view of the quality of the design. However the design methods are very different: analytical methods and expert systems can be integrated [8.], knowledge base and metaheuristic can be combined [9.], decomposition based programming methods can be used [10.], but the correct mathematical modelling is very important from the point of view of optimisation results.

The detailed description of materials handling systems is a difficult task that heavily affects the efficiency and the strategic decision that many production and service enterprises have to make every time they acquire a new materials handlings (or logistic) system or modify or improve an existing one [11-13.]. This work is a part of the research activity of the "Improving the quality of higher education on the basis of the development of centres of excellence in strategic research filed of the University of Miskolc".

1. Mathematical modelling

The products enter the system from the warehouse unit. The products enter the sorting system by the aid of a materials handling machine (e.g. fork lift or robot) and are distributed onto the certain classifier buffer in compliance with the distribution strategy. The classifier buffers are in fact gathering paths with the help of which different distribution strategies can be formed according to the task. When designing this materials handling system, we have to take the operation costs in addition to the investment costs for the definition of the optimal system parameters into consideration. As the investment cost of automatic machines can be extremely high a significant part of the objective function depends on the number and type of this machine.

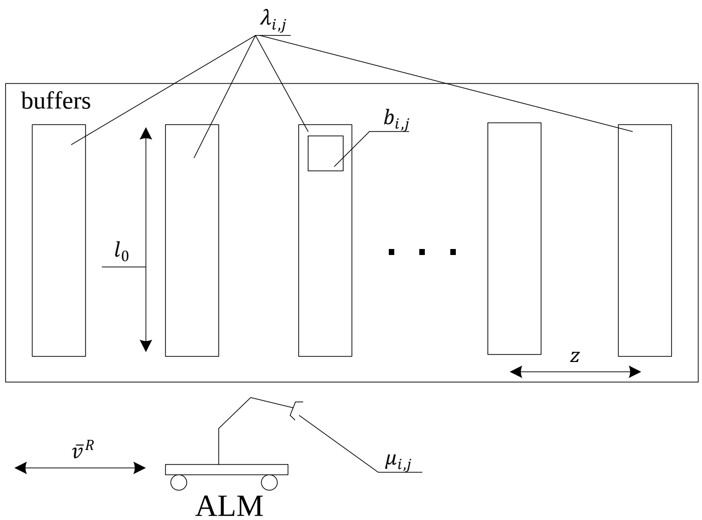

Figure 1: Model of the automatic loading system

Table 1. Input parameters I.

Input parameter	Explanation for the input parameters
$\lambda_{i,j}$	Arrival intensity of products on the i^{th} buffer in the j^{th} service program.
f	Number of buffers in the system.
$b_{i,j}$	Length of the products on the i^{th} buffer in the j^{th} service program.
z	Distance between buffers.
p_j^{SP}	Probability of the j^{th} service program.
$x_{i,j}$	Number of products to be handled within the frame of one cycle of the j^{th} service program on the i^{th} buffer.
${}^{I}c^{h}$	Specific investment cost of the materials handling unit of the automatic loading machine in the case of a predefined materials handling intensity.
${}^{I}c^{t}$	Specific investment cost of the transport unit of the automatic loading machine in the case of a predefined speed.
${}^{I}c^{b}$	Specific investment cost of the buffer in the case of a predefined length.
$\alpha, \beta, \chi, \delta, \phi, \varphi$	Coefficient of the different functions.
w	Number of service programs.

Table 2. Input parameters II.

Input parameter	Explanation for the input parameters
${}^{O}c^{h}$	Specific operation cost of the materials handling unit of the automatic loading machine in the case of a predefined materials handling intensity.
${}^{O}c^{t}$	Specific operation cost of the transport unit of the automatic loading machine in the case of a predefined speed.
${}^{O}c^{b}$	Specific operation cost of the buffer.
${}^{I}c_{lt}^{h}$	Specific additional cost of the materials handling unit of the automatic loading machine in the case of a lifetime unit.
${}^{I}c_{lt}^{t}$	Specific additional cost of the transportation unit of the automatic loading machine in the case of a lifetime unit.
${}^{I}c_{lt}^{b}$	Specific additional cost of the buffer in the case of a lifetime unit.
T_0	Time interval of the optimisation.

The total investment cost includes the investment cost of the materials handling and transportation unit of the automatic loading machine and the investment cost of the buffer.

$$ {}^{I}C = {}^{I}C^{h} + {}^{I}C^{t} + {}^{I}C^{b} \quad (1) $$

where ${}^{I}C$ is the investment cost of the analysed system,
${}^{I}C^{h}$ is the investment cost of the materials handling unit of the automatic loading machine,
${}^{I}C^{t}$ is the investment cost of the transportation unit of the automatic loading machine,
${}^{I}C^{b}$ is the investment cost of the buffers.

The investment cost of the materials handling unit depends on the required loading intensity and the lifetime. In this form the service strategy is not taken into consideration.

$$ {}^{I}C^{h} = {}^{I}c^{h} \cdot \left(\mu_{max}^{opt.}\right)^{\alpha} + {}^{I}c_{lt}^{h} \cdot (N)^{\beta} \quad (2) $$

where $\mu_{max}^{opt.}$ is the upper limit of required loading intensity,
N is the lifetime of the materials handling unit of the automatic loading machine.

The investment cost of the transportation unit of the automatic loading machine depends on the required speed and lifecycle.

$$ {}^{I}C^{t} = {}^{I}c^{t} \cdot \left(v_{opt\,.max}^{R}\right)^{\chi} + {}^{I}c_{lt}^{t} \cdot (H)^{\delta} \quad (3) $$

where $v_{opt\,.max}^{R}$ is the upper limit of required speed,
H is the lifetime of the transportation unit of the automatic loading machine.

The investment cost of the buffers depends on the number of buffers, length and lifetime of buffers.

$$ {}^{I}C^{b} = f \cdot \left({}^{I}c^{b} \cdot \left(l_{0}^{opt\,.max}\right)^{\varphi} + {}^{I}c_{lt}^{b} \cdot (\Omega) \right) \quad (4) $$

where $l_0^{opt.max}$ is the upper limit of the length of the buffers,
Ω lifetime of the buffers.

The operation cost of the materials handling unit of the automatic loading machine for a time period depends on the average required loading intensity and the specific maintenance cost.

$$ {}^{O}C_{T_0}^{h} = T_0 \cdot \bar{\mu} \cdot {}^{O}c^{h} \cdot c_m^h \qquad (5) $$

where $\bar{\mu}$ is the average required loading intensity calculated from the probability of service programs and the numbers of loaded products,
c_m^h is the specific maintenance cost of the materials handling unit of the automatic loading machine.

$$ \bar{\mu} = \sum_{j=1}^{w} p_j^{SP} \sum_{i=1}^{f} \left(\frac{\frac{l_{i,j}^0 / b_{i,j}}{1-\lambda_{i,j}/\mu_{i,j}}}{\sum_{h=1}^{f} \frac{l_{i,j}^0 / b_{h,j}}{1-\lambda_{h,j}/\mu_{h,j}}} \cdot \mu_{i,j} \right) \ and \ \sum_{j=1}^{w} p_j^{SP} = 1 \qquad (6) $$

The operation cost of the transportation unit of the automatic loading machine depends on the specific operation cost, the length of the transportation route within the frame of one service program, speed, number of cycles and maintenance cost.

$$ {}^{O}C_{T_0}^{t} = \frac{T_0}{\bar{t}_0} \cdot \bar{v}^R \cdot {}^{O}c^{t} \cdot s \cdot c_m^t \qquad (7) $$

where s is the route of the automatic loading machine within the frame of one cycle,
$\bar{v}^R$ is the average speed of the automatic loading machine depending on the probability of service strategies,
$\bar{t}_0$ average cycle time of service programs,
c_m^h specific maintenance cost of the transport unit of the automatic loading machine.

$$ \bar{v}^R = \sum_{j=1}^{w} p_j^{SP} \cdot \left(\bar{v}_j^R\right)^{\eta} \ and \ \bar{t}_0 = \sum_{j=1}^{w} p_j^{SP} \cdot \left(\sum_{i=1}^{f} \frac{l_{i,j}^0}{b_{i,j} \cdot \left(\mu_{i,j} - \lambda_{i,j}\right)} + \frac{2 \cdot (f-1) \cdot z}{v_j^R} \right) \qquad (8) $$

The operation cost of the buffers depends on the specific operation and maintenance cost and the interval of operation.

$$ {}^{O}C_{T_0}^{b} = {}^{O}c^{b} \cdot T_0 \cdot f \cdot c_m^b \qquad (9) $$

The probability of service programs is given. The investment cost for one cycle as amortisation cost can be defined for the two main units of the automatic loading machine and buffer.

$$ {}^{I}C_{T_0}^{h} = \frac{{}^{I}c^{h} \cdot \left(\mu_{max}^{opt.}\right)^{\alpha} + {}^{I}c_{m}^{h} \cdot (N)^{\beta}}{\frac{N}{\bar{\mu} \cdot T_0}} \tag{10} $$

$$ {}^{I}C_{T_0}^{t} = \frac{{}^{I}c^{t} \cdot \left(v_{opt.max}^{R}\right)^{\chi} + {}^{I}c_{m}^{t} \cdot (H)^{\delta}}{\frac{H}{T_0/(\bar{t}_0 \cdot s)}} \tag{11} $$

$$ {}^{I}C_{T_0}^{b} = f \cdot \frac{{}^{I}c^{b} \cdot \left(l_{0}^{opt.max}\right)^{\phi} + {}^{I}c_{m}^{b} \cdot (\Omega)^{\varphi}}{\frac{\Omega}{T_0}} \tag{12} $$

According to the above mentioned costs, the total operation cost can be calculated as follows.

$$ {}^{O}C = {}^{I}C_{T_0}^{h} + {}^{I}C_{T_0}^{t} + {}^{I}C_{T_0}^{b} + {}^{I}C_{T_0}^{h} + {}^{I}C_{T_0}^{t} + {}^{I}C_{T_0}^{b} \tag{13} $$

Table 3. Constraints

Contraints	**Explanation for the constraints**
$l_{0,min} \leq l_0 \leq l_{0,max}$	The length of the buffers must be between the lower and upper bound of the predefined length.
$\mu_{0,min} \leq \mu_0 \leq \mu_{0,max}$	The loading intensity must be between the lower and upper bound.
$H_{min} \leq H \leq H_{max}$ $N_{min} \leq N \leq N_{max}$ $\Omega_{min} \leq \Omega \leq \Omega_{max}$	The lifetime of the different units must be between the predefined lower and upper bound of lifetime.

However the above mentioned cost functions are convex, so the minimisation problem should be solved with derivation, but because of the complex relationship among system parameters (length of buffer, loading intensity, speed) the optimisation problem (cost minimisation) can be solved by the aid of heuristic methods.

2. Systemparameters

The aim of this chapter is to define the connection among length of buffers, loading intensity of the handling unit of the automatic loading machine and the speed of the transport unit of the automatic loading machine. By the aid of this formula it is possible to define one possible set of system parameters depending on the arrival intensity of the products for each cycle and service program, length of the products, number of buffers, distances between buffers for each service strategy. It is possible to define six possible service strategies:

- loading time is proportional to arrival intensity,
- the loading time is the same by each buffer,
- the same amount of products is loaded by each buffer within the frame of one cycle,
- the amount of loaded products for each buffer and service cycle is proportional to the arrival intensity,
- the loading intensity is proportional to the arrival intensity,
- the loading time is inversely proportional to the time needed to empty the buffer.

As an example let us show the connection among system parameters in the case of strategy A. The loading time is proportional to arrival intensity. The cycle time is the sum of the loading time by each buffers and the transition time among buffers. The number of arrived products is the multiplication of the arrival time and arrival intensity.

$$t_{i,j} \cdot \mu_{i,j} = \lambda_{i,j} \cdot \left(t_j^A + \sum_{h=1}^{f} t_{h,j} \right) \; and \; \frac{t_{i,j}}{\lambda_{i,j}} = const. \tag{14}$$

where $t_{i,j}$ is the time spent by the automatic loading machine at the i[th] buffer in the j[th] service program,

t_j^A is the transition time among buffers,

$\mu_{i,j}$ is the loading intensity of the automatic loading machine at the i[th] buffer in the j[th] service program.

The transition time of the automatic loading vehicle among buffers depends on the speed of the machine, distance among buffers and number of buffers.

$$t_j^A = \frac{2 \cdot (f-1) \cdot z}{v_j^R} \tag{15}$$

where v_j^R is the speed of the transportation unit of the automatic loading machine in the j[th] service program.

The required loading intensity can be calculated from the equation (14).

$$\mu_{i,j} = \frac{\lambda_{i,j}}{t_{i,j}} \cdot \sum_{h=1}^{f} t_{h,j} + \frac{t_j^A}{t_{ij}} \cdot \lambda_{i,j} \tag{16}$$

The loading time is proportional to the arrival intensity, so the equation (15) can be simplified.

$$\mu_{i,j} = \sum_{h=1}^{f} \lambda_{h,j} + \frac{\lambda_{i,j}}{t_{ij}} \cdot t_j^A \tag{17}$$

The loading intensity must be the same in each service strategy because of the speed of the transport unit of the automatic loading machine and the proportion of the arrival intensity and loading time in each service program are constant. It means, that the equation (17) can be modified.

$$\mu_j = \sum_{h=1}^{f} \lambda_{h,j} + \frac{\lambda_{i,j}}{t_{ij}} \cdot \frac{2 \cdot (f-1) \cdot z}{v_j^R} \tag{18}$$

If the buffer is empty, then the loading cycle of the automatic loading machine will be finished, so the number of loaded products can be described as the sum of an infinite geometric series, where the ratio is the proportion of arrival and loading intensity.

$$x_{i,j} = \frac{k_{i,j}}{1 - \frac{\lambda_{i,j}}{\mu_{i,j}}} \tag{19}$$

where $k_{i,j}$ is the number of the products on the i^th^ buffer in the j^th^ service program,

$x_{i,j}$ is the number of loaded products on the i^th^ buffer in the j^th^ service program.

The length of the buffers can be calculated depending on the loading time for each buffers and service programs.

$$l_0 = \max_{j=1\dots w}\left(\max_{i=1\dots f}\left(\mu_j \cdot t_{i,j} \cdot b_{i,j} \cdot \left(1 - \frac{\lambda_{i,j}}{\mu_j}\right)\right)\right) \tag{20}$$

where l_0 is the required length of buffers.

In the case of other strategies this calculation method can be more complicated. If the loading time is inversely proportional to the time needed to empty the buffer, then the equation to define the length of the buffers is the following:

$$l_{i,j}^0 = \frac{\lambda_{i,j} \cdot 2 \cdot (f-1) \cdot z}{v_j^R \cdot \left\{\frac{1}{b_{i,j} \cdot \left(1 - \frac{\lambda_{i,j}}{\mu_{i,j}}\right)} - \frac{\mu_{i,j}}{b_{i,j} \cdot \left(\mu_{i,j} - \lambda_{i,j}\right)} \cdot \sum_{h=1}^{f} \frac{\lambda_{h,j}}{\mu_{h,j}}\right\}} \tag{21}$$

3. Consequences

Today the materials handling systems are gaining more and more importance, as being flexible and reliable, they significantly increase the efficiency of materials handling and reduce the waiting time. As the investment and operation costs of automatic materials handling machines are rather high, it is a very important milestone of the design to optimise the parameters of the system and resources. Within the frame of this paper the author describe one possible mathematical model of a materials handling subsystem of a warehouse (part loading of products). The cost based objective function was defined. However this cost function -including operation and investment costs- is a convex function, but because of the complex connection among system parameters (length of the buffers, speed of the transportation and loading intensity of the handling units of the automatic loading machine) the minimisation of the cost is possible with heuristic methods.

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