SELECTED PROBLEMS OF INVENTORY MANAGEMENT IN ENTERPRISES

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Abstract: Identification of optimal solutions for order quantity reduces costs of transactions and ensures successful dealing with the requests. Proper planning, combined with lower cost of transactions based on current inventory level, storage surface area and lead time impacts on profitability in the enterprise, thus on its position in the market. The present paper presents considerations which provide basis for performance of inventory management goals through optimization of order quantity.

Keywords: stock, optimal order quantity, transaction costs, dynamic programming.

1. Introduction to Inventory Management

Stock is indispensable element of enterprise functioning and it is defined as ‘a value or amount of raw materials, components, goods, semi-finished products and final products, kept or stored in order to be consumed if the need arises’ [1.]. Proper inventory management is of critical importance to business entities since inventory generates costs as well as affects improvement in customer service level [2.].
Inventory, which is an element of current assets in companies, appears in the form of raw materials, semi-finished products or final products which allow for: [3.]

- improved customer service level,
- enhanced economy of scale in production,
- measurable results connected with large batches for transport and procurement,
- protection from fluctuations in prices, exchange rates as well as supply and demand and other factors difficult to be forecast.

Business activity forces enterprises to keep a particular amount of inventory. Business entities strive for formation of inventory levels at a suitable level, which enables them to fully perform the assigned tasks.
The most essential factors that impact on optimal amount of stock in enterprises include: [4.]

- inventory management,
- demand for products,
- the used methods of production, e.g. piece production, batch production etc.,
- type of demand for a particular stock (dependent, independent),
- factors resulting from market demand (should goods be purchased in advance or at a particular time?),
- factors of e.g. production which affect optimal order (delivery) quantity.

The purpose of maintaining the inventory in companies is ability to meet customers' expectations concerning the type, quantity and time of deliveries combined with use of production capacities and inventory cost reduction [5.]. Control over inventory level is connected with such planning, execution and control of suitable amount and structure of stock which allows for adjustment of order quantity to the request and brings opportunities of minimization of costs of warehousing and stock [6.]. Therefore, a concept of economic order quantity must be explained as 'amount/quantity of goods in the given delivery which ensures minimization of total inventory costs' [7.]. The method of determination of optimal order quantity will be presented in next subchapter.

2. Fundamental Issues of Dynamic Programming

The investigated enterprise orders a particular quantity of requested semi-finished products every week. On the basis of the planned values of use of the semi-finished products and at the given unit costs of transactions at the beginning of each week, a plan of semi-finished products purchase for a period of month should be presented so that transaction costs are as low as possible.
The solution for this problem is based on the following assumptions: [8.]

- initial inventory for the semi-finished products is known,
- inventory is approaching zero at the end of month,
- semi-finished products delivery is executed immediately,
- total capacity of all warehouses is limited,
- inventory in warehouses decreases at the same rate.

In consideration of the assumptions, the given decision problem can be solved using dynamic software, developed first by R.E. Bellman [9.].
In order to simplify the task, the following symbols are introduced: [8.]

p_i- planned consumption of semi-finished products in *i* week,
c_i- unit transaction costs at the beginning of *i* week,
z_i- inventory level at the end of *i* week,
a_i- level of planned purchase of semi-finished products at the beginning of *i* week,
x_i- decision variables which reflect inventory level at the beginning of *i* week, after delivery of a_i to the warehouse,
z- maximal inventory level.

On the basis of the assumption and the introduced symbols, the problem can be graphically presented as in Fig. 1.

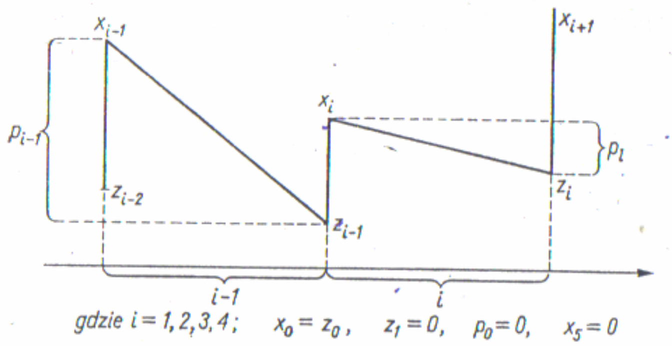

Figure 1. Graphic presentation of dynamic programming problem
Source: [8.]

The figure is then used for determination of the conditions of internal concordance, which should be achieved by means of decision variables x_i. For *i* week it is assumed that: [8.]

$$x_i \geq p_i \quad i \quad x_i \geq z_{i-1} = x_{i-1} - p_{i-1} \tag{1}$$

thus

$$x_i \geq \max(p_i, x_{i-1} - p_{i-1}) \tag{2}$$

where

i=1, 2, 3, 4 and
$x_0=z_0$; $p_0=0$.

This causes that inventory level at the beginning of *i* week is not lower than:

- the expected consumption of semi-finished products in this week,
- difference in inventory level at the beginning of the previous week,
- the expected consumption in the previous week.

"Properly managed inventory impacts on customers, suppliers and main functional departments in organization" [10.]
It is also remarkable that: [8.]

$$x_i \leq z \quad i \quad x_{i+1} \geq z_i = x_i - p_i \tag{3}$$

hence

$$x_i \leq z \quad i \quad x_i \leq x_{i+1} + p_i \tag{4}$$

thus

$$x_i \leq \min(z, x_{i+1} + p_i) \tag{5}$$

where

i=1, 2, 3, 4 and
$x_5=0$.

As results from the abovementioned formulae, inventory level at the beginning of *i* week is not greater than:

- maximal inventory in next week,
- total of inventory levels,
- the expected consumption of semi-finished products in *i* week.

"Inventory level of semi-finished products also must fulfill the same conditions as levels of other inventories" [11.].
In consideration of the formulae (1,2) and (3,4,5) the limitations to decision variable i.e. inventory level at the beginning of *i* week are given by: [8.]

$$\max(p_i, x_{i-1} - p_{i-1}) \leq x_i \leq \min(z, x_{i+1} + p_i) \tag{6}$$

where

i=1,2,3,4 and
$x_0=z_o$ and $p_o=0$ i $x_5=0$.

"Inventory management is of significant importance to each company" [12.].
Using Bellman's principles of optimization and with the assumption that $f_{4,3,2,1}$ is minimal total costs of transaction in first, second, third and fourth week and that k_i denotes transaction costs at the beginning of *i* week, the following formulae are obtained: [8.]

$$f_{4,3,2,1} = \min_{x_i}\left[k_1(x_1) + f_{4,3,2,1}\right] \tag{7}$$

$$f_{4,3,2} = \min_{x_2}\left[k_2(x_2) + f_{4,3}\right] \tag{8}$$

$$f_{4,3} = \min_{x_3}\left[k_3(x_3) + f_4\right] \tag{9}$$

$$f_4 = \min_{x_4}\left[k_4(x_4)\right] \tag{10}$$

where

x_i=1,2,3,4 realize relation 3.

The use of inventory management problem is presented in the example below.

3. Application of Dynamic Programming for Determination of Optimal Order Quantity - Case Study

Having the data given in Tab. 1, presenting the expected consumption of semi-finished good and transaction costs at the beginning of *i* week, the ordered quantities for raw-materials in the beginning of each week should be determined so that total transaction costs should be as low as possible.
It is assumed that: [13.]

- total capacity of warehouses amounts to 10 units,
- initial inventory amounts to 1 unit,
- final inventory amounts to 0 units.

Table 1. Quantity of predicted semi-finished products usage and transaction costs at the beginning of i week

Week (i)	**I**	**II**	**III**	**IV**
Raw material consumption (p_i)	2	3	4	7
Transaction costs (c_i)	7	6	3	4

Source: "author's own calculations"

The calculations were started with fourth quarter of the year.
As results from assumptions, z_4=0 thus x_4-p_4=0, hence: x_4=p_4=7 (Table 1.), which causes that the level of semi-finished products at the beginning of week 4 corresponds to the expected consumption of semi-finished products in this week and amounts to 7 units. The transaction costs connected with purchasing of a_4 units of semi-finished products amount to: [13.]

$$K_4(X_4)= c_4a_4=c_4(x_4-z_3)=c_4[x_4-(x_3-p_3)],$$

where

c_4=4, x_4=7, p_3=4.

hence

$$f_4 = \min_{x_4} K_4(x_4) = K_4(7) = 44 - 4_{x_3} \tag{11}$$

Relation (6) was used for determination of variability range for decision variable x_3:

$$\max(4, x_2 - 3) \leq x_3 \leq \min(10, 7+4) \tag{12}$$

thus:

$$\max(4, x_2 - 3) \le x_3 \le 10. \tag{13}$$

This causes that:

$$x_i \le 10 \tag{14}$$

$$i=1,2,3$$

thus:

$$4 \le x_3 \le 10 \tag{15}$$

During transaction cost analysis in third and fourth week the value given by the following formula should also be calculated:

$$f_{4,3} = \min_{4 \le x_3 \le 10} \left[K_3(x_3) + f_4 \right]. \tag{16}$$

Transaction costs related to the purchase of a_3 units of semi-finished products are given by:

$$K_3(X_3) = c_3 a_3 = c_3(x_3 - z_2) = c_3[x_3 - (x_2 - p_2)],$$

where:

$c_3=3$, $p_2=3$

thus:

$K_3(X_3)=3x_3-3x_2+9$

and

$$f_{4,3} = \min_{4 \le x_3 \le 10} \left[-x_3 - 3x_2 + 53 \right] \tag{17}$$

For x3, function $y_1 = -x_3 - 3\ x_2 + 53$ is decreasing, thus in the range of [4.,10.] it reaches lowest value for $x_3=10$. As results from this fact:

$$f_{4,3} = -3_{x_2} + 43 \tag{18}$$

Similarly to the previous case, variability range for decision variable x_2 is given by (6): [13.]

$$\max(3, x_1 - 2) \le x_2 \le \min(10, 10+3) \tag{19}$$

hence:

$$\max(3, x_1 - 2) \le x_2 \le 10 \tag{20}$$

Another stage is analysis of transaction costs in second, third and fourth week:

$$f_{4,3,2} = \min_{\max(3, x_1 - 2) \le x_2 \le 10} \left[K_2(x_2) + f_{4,3} \right] \tag{21}$$

where:

$$K_2(X_2) = c_2 a_2 = c_2(x_2 - z_1) = c_2[x_2 - (x_1 - p_1)]$$

where $c_2=6$, $p_1=2$

which results in:

$$K_2(X_2) = 6x_2 - 6x_1 + 12$$

hence:

$$f_{4,3,2} = \min_{\max(3, x_1 - 2) \le x_2 \le 10} (3x_2 - 6x_1 + 55) \tag{22}$$

Due to x_2 value within the range of [max $(3,x_1-2);10$] function $y_2= 3x_2-6x_1+55$ is increasing whereas minimal value is reached for $r= \max(3,x_1-2)$ thus:

$$f_{4,3,2} = 3r - 6x_1 + 55 \tag{23}$$

where

$$r= \max(3,x_1-2).$$

As results from relation (6):

$$\max(2,1) \le x_1 \le 10 \tag{24}$$

The following two cases should be considered: [13.]

1) If:

$$9 \langle x_1 \le 10 \tag{25}$$

then $r = x_1-2$.

2) If:

$$2 \langle x_1 \le 9 \tag{26}$$

then $r=3$.

For the first case:

$$f_{4,3,2} = -3x_1 + 49 \tag{27}$$

where:

$$9 \langle x_1 \le 10 \tag{28}$$

For the second case:

$$f_{4,3,2} = -6x_1 + 64 \tag{29}$$

where:

$$2 \langle x_1 \le 9 . \tag{30}$$

Another stage is to analyse transaction costs in first week with consideration of transaction costs in other weeks. Minimal total costs of transactions amount to: [13.]

$$f_{4,3,2,1} = \min\left[f^{(1)}_{4,3,2,1}; f^{(2)}_{4,3,2,1}\right] \tag{31}$$

where:

$$f_{4,3,2,1} = \min_{9\langle x_1 \le 10}\left[K_1(x_1) + f^{(1)}_{4,3,2}\right] \tag{32}$$

$$f_{4,3,2,1} = \min_{2\langle x_1 \le 9}\left[K_1(x_1) + f^{(2)}_{4,3,2}\right] \tag{33}$$

Purchase costs for transaction a_i are given by:

$$K_1(X_1)= c_1a_1=c_1(x_1-z_0)$$

where:

$c_1=7$, $z_0=1$

hence:

$$K_1(X)=7x_1-7$$

thus:

$$f^{(1)}_{4,3,2,1} = \min_{9\langle x_1 \leq 10}\left(7x_1 - 7 - 3x_1 + 49\right) = \min_{9\langle x_1 \leq 10}\left(4x_1 + 42\right) \tag{34}$$

$$f^{(2)}_{4,3,2,1} = \min_{2\langle x_1 \leq 9}\left(7x_1 - 7 - 6x_1 + 64\right) = \min_{2\langle x_1 \leq 9}\left(x_1 + 57\right) \tag{35}$$

In both analysed ranges, functions $y_3 = 4x_1+42$ and $y_4 = x_1+57$ are increasing, hence:

$$f^{(1)}_{4,3,2,1} = 36 + 42 = 78 \tag{36}$$

$$f^{(2)}_{4,3,2,1} = 2 + 57 = 59 \tag{37}$$

thus: $x_2=2$ and $f_{4,3,2,1}=59$, which gives minimal transaction costs at the level of 59 monetary units.

Table 2. Results from solving case study with use of Dynamic Programming

i	**x_i**	**p_i**	**z_{i-1}**	**a_1**	**c_1**	**a_1c_1**
1	2	2	1	1	7	7
2	3	3	0	3	6	18
3	10	4	0	10	3	30
4	7	7	6	1	4	4
						$\sum_{i=1}^{4} a_i c_i = 59$

Source: "author's own calculations"

Purchase quantity for semi-finished products at the beginning of each week is presented in Table 2. while minimal costs of transaction amount to 59 monetary units.

4. Summary

Dynamic programming allows for planning of the ordered quantity so that transaction costs connected with procurement are possibly lowest. Method of purchase optimization employed in enterprises can become a source of savings which can be ploughed back in other profit-generating areas of business operation. High levels of inventory are used by many companies to hide a number of problems that occur during their functioning on the market. Companies may have sound and acceptable reasons for holding stock but some may also use high levels of inventory to protect themselves from those problems. They also try to change their attitude towards inventory management. Because of this changing approach to inventory responsibility, the traditional methods of inventory planning are now becoming less applicable for many companies. This is why companies apply such methods of describing inventory levels such as dynamic programming, especially when inventory requirements are determined in relation to the type of product.

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